# Contextuality and indistinguishability

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Quantum Contextuality in Quantum Mechanics and Beyond

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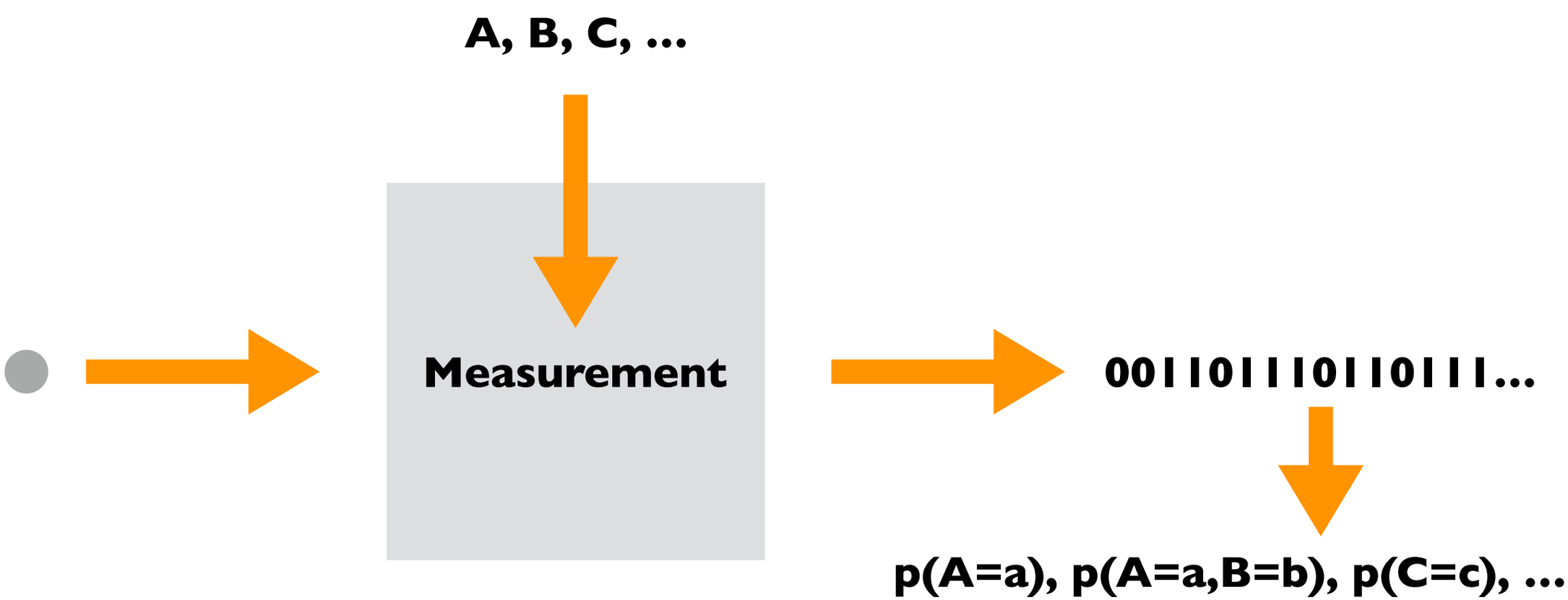
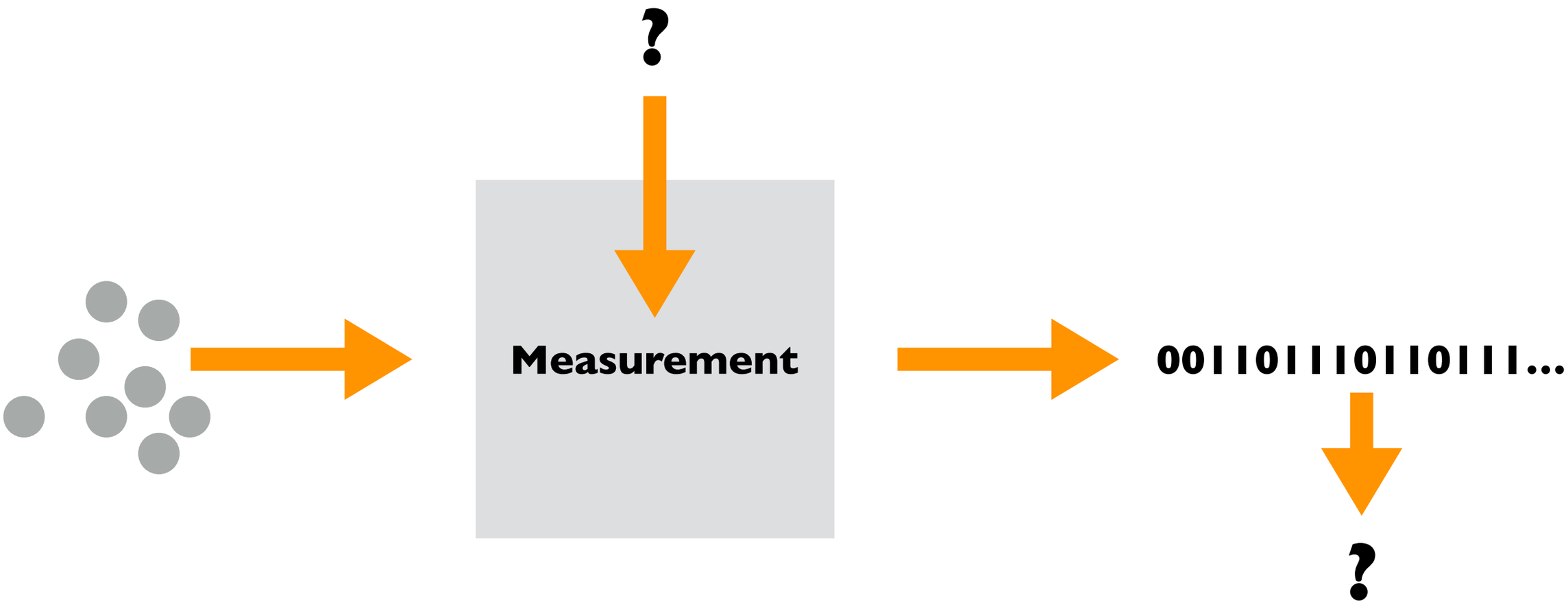
# Contextuality - single system




**Goal: lack of explanation within NCHV theories**

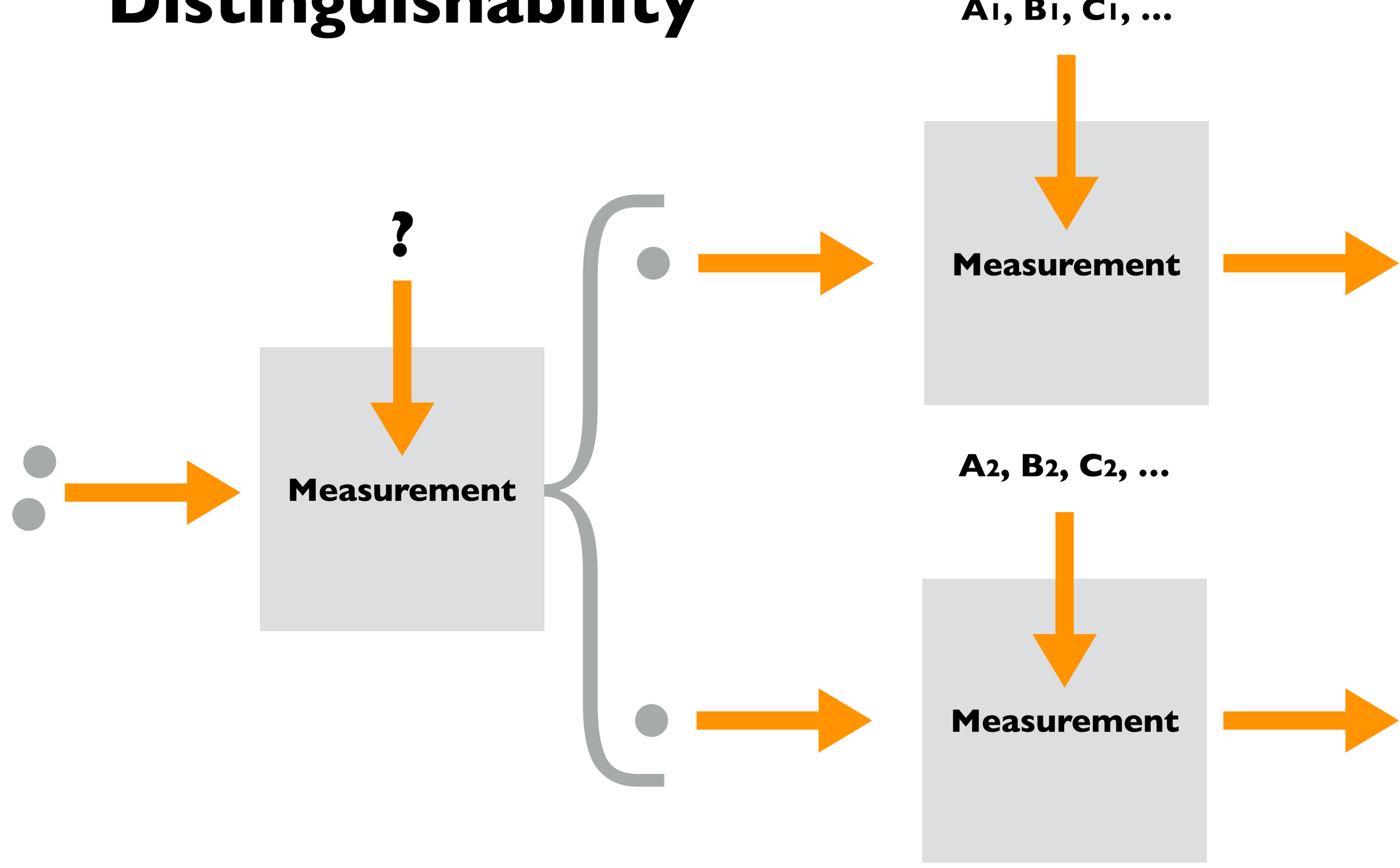
# Contextuality - many systems




**Goals: - find possible measurements**

**- lack of explanation within NCHV theories**

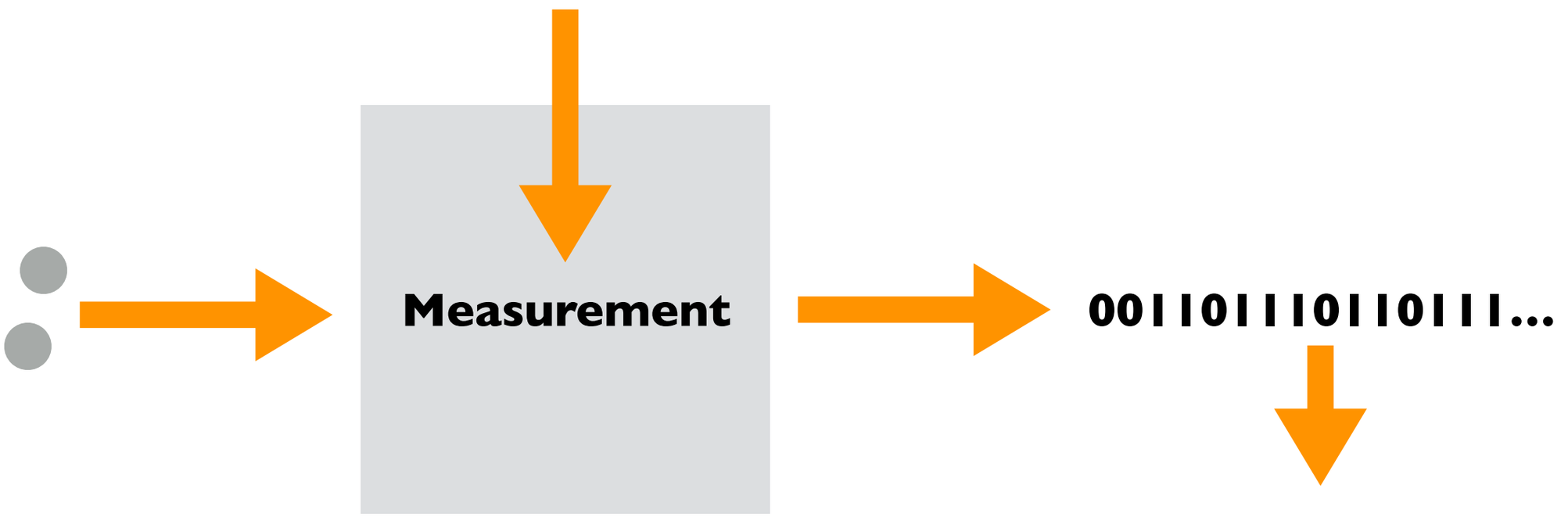
# Distinguishability




**Bell scenario, ..., two independent single-system experiments**

# Indistinguishability

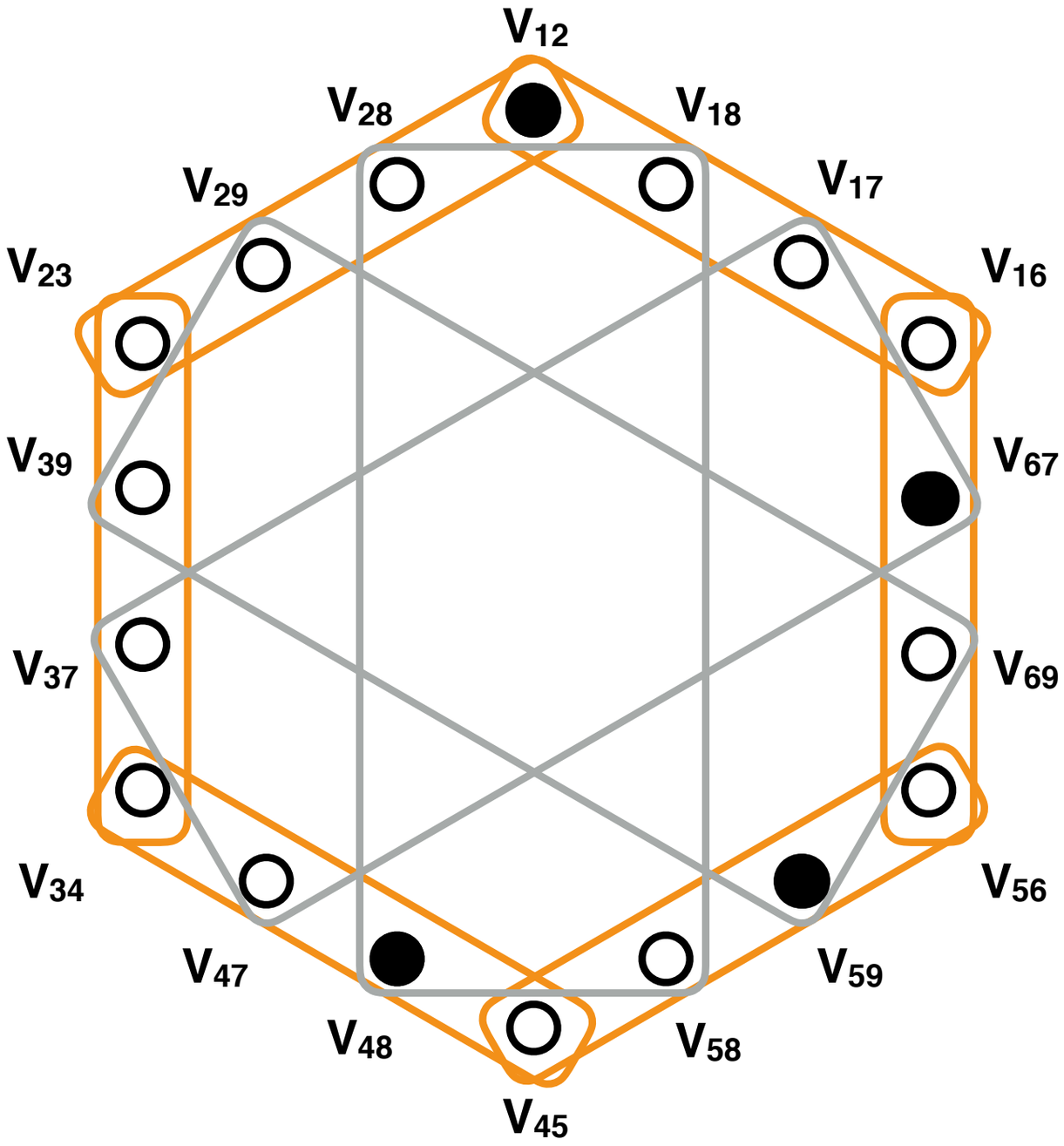
**limited set of measurements**




**Example:** $A_1 = 0,1 \quad A_2 = 0,1$

**But we can only measure:** $A_1 + A_2 = 0, 1, 2$

# Contextuality as lack of conservation




$V_{12} = (1,0,0,0)$

$V_{28} = (0,0,0,1)$

$V_{29} = (0,1,1,0)$

$V_{23} = (0,1,-1,0)$

$V_{39} = (1,0,0,1)$

$V_{37} = (1,1,1,-1)$

$V_{34} = (-1,1,1,1)$

$V_{47} = (1,1,-1,1)$

$V_{48} = (1,0,1,0)$

$V_{45} = (0,1,0,-1)$

$V_{58} = (1,0,-1,0)$

$V_{59} = (1,-1,1,-1)$

$V_{56} = (1,1,1,1)$

$V_{69} = (1,1,-1,-1)$

$V_{67} = (1,-1,0,0)$

$V_{16} = (0,0,1,-1)$

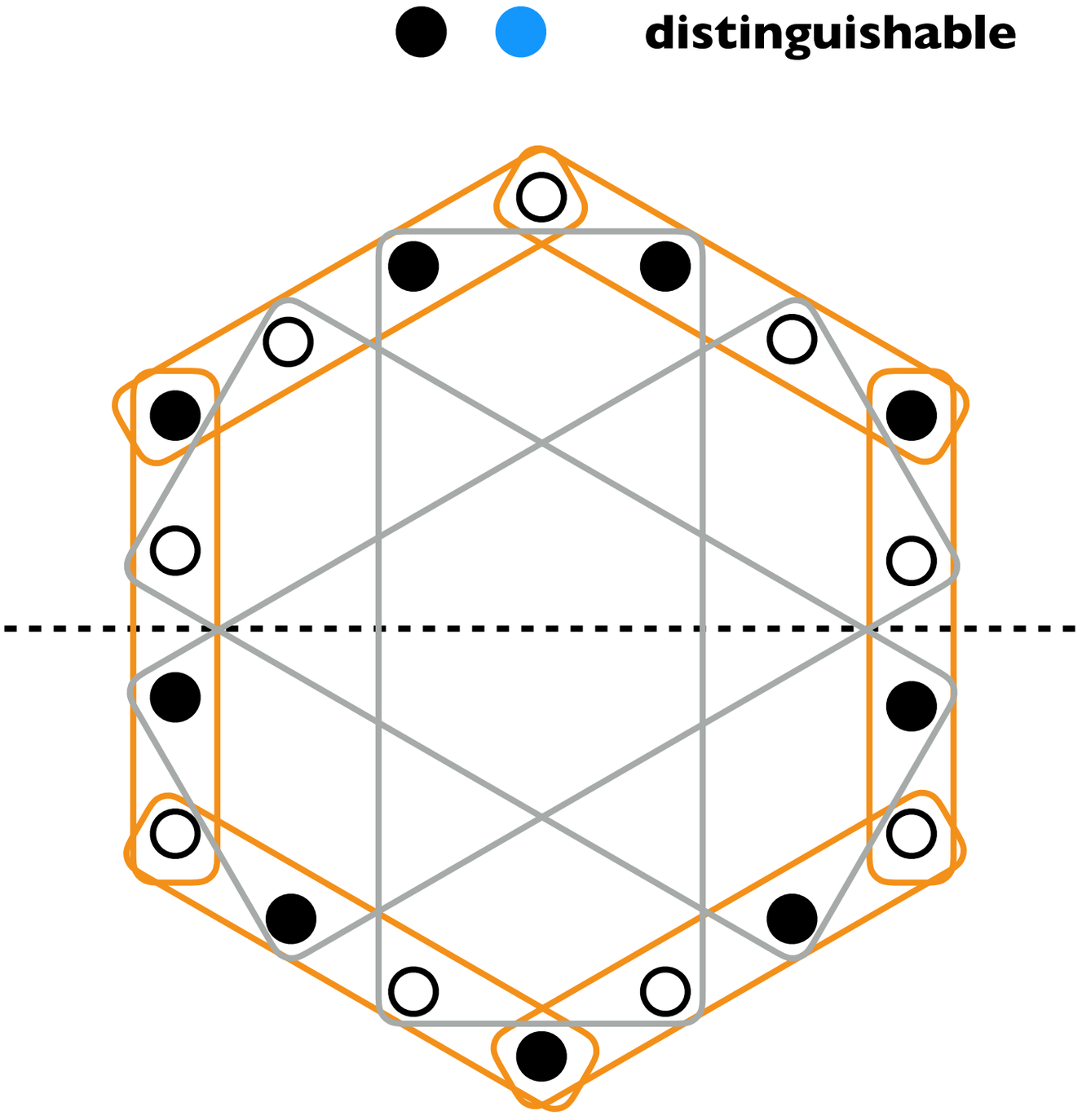
$V_{17} = (0,0,1,1)$

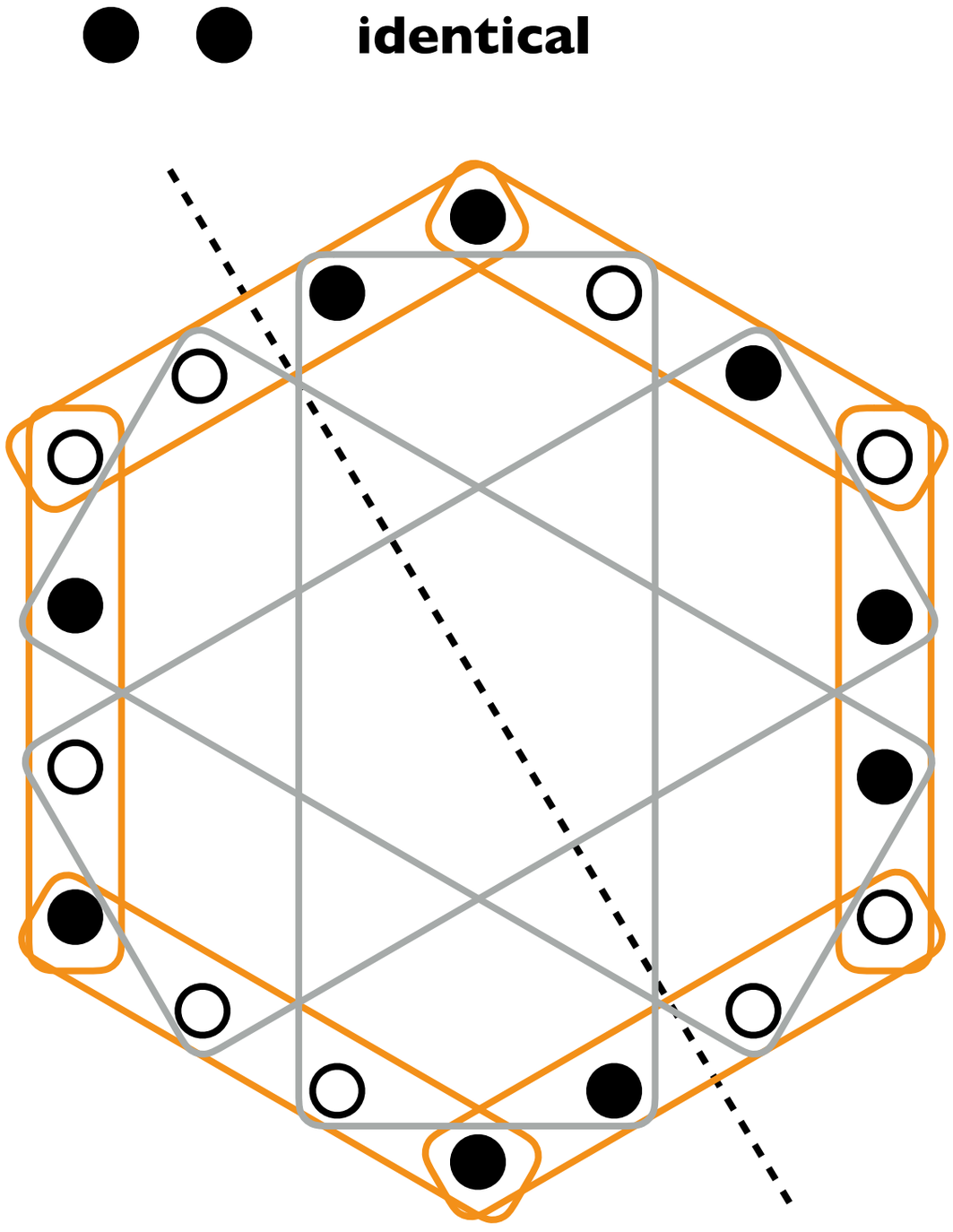
$V_{18} = (0,1,0,0)$

A. Cabello, J. M. Estebaranz, and G. Garcia Alcaine, Phys. Lett. A **212**, 183 (1996)

A. Cabello, Phys. Rev. Lett. **101**, 210401 (2008)

# Two particles



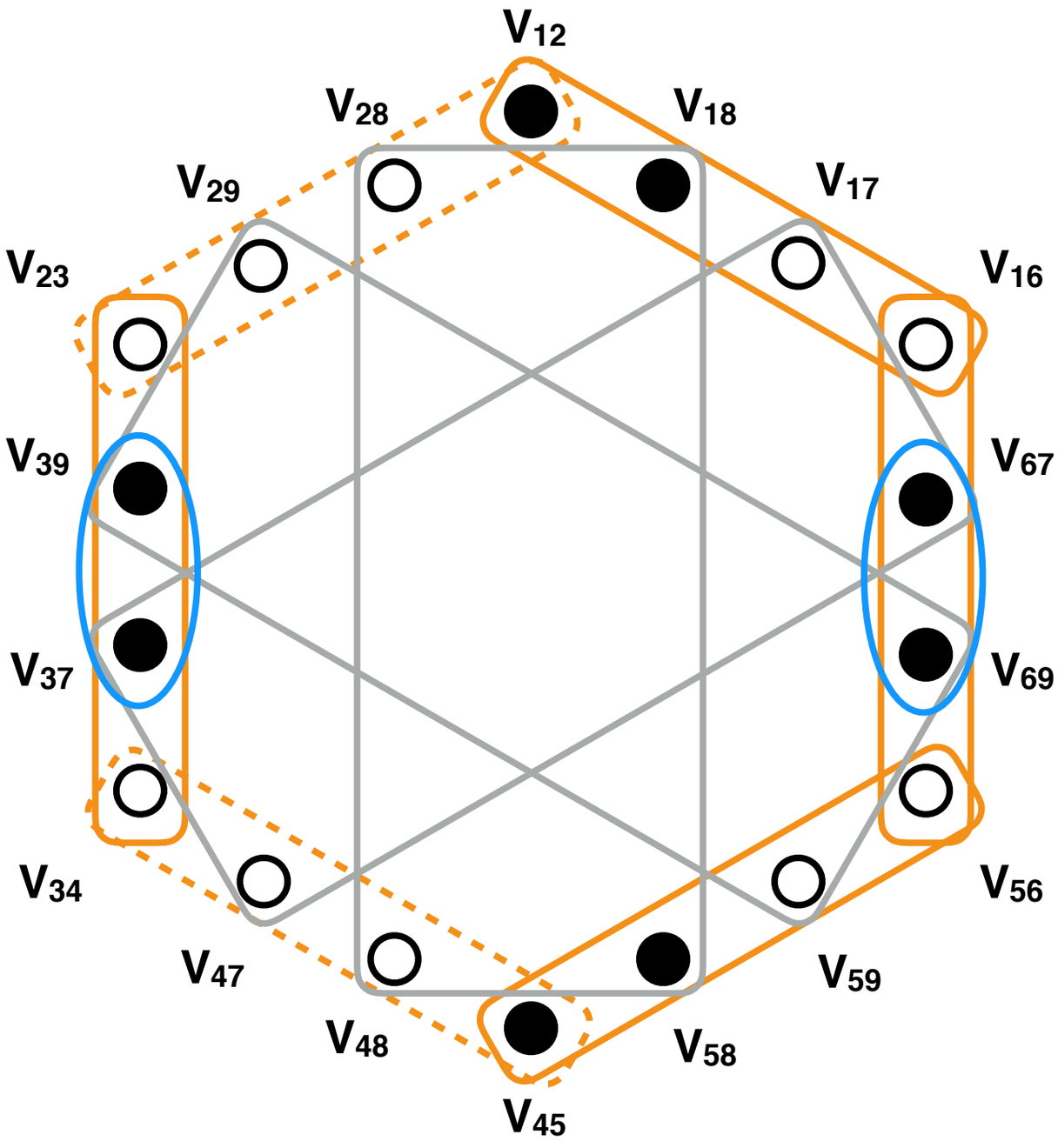

PK, Phys. Rev. A 95, 012133 (2017)

# State independent contextually Fermions vs Bosons

| N | Fermions | Bosons |
|---|---|---|
| 0 | No | No |
| I | Yes | Yes |
| 2 | No | No |
| 3 | Yes | Yes |
| 4 | No | No |
| 5 | - | Yes |
| ... | - | ... |

**Parity argument for bosons - M. Karczewski**

# Hardy-type contextuality - fermions

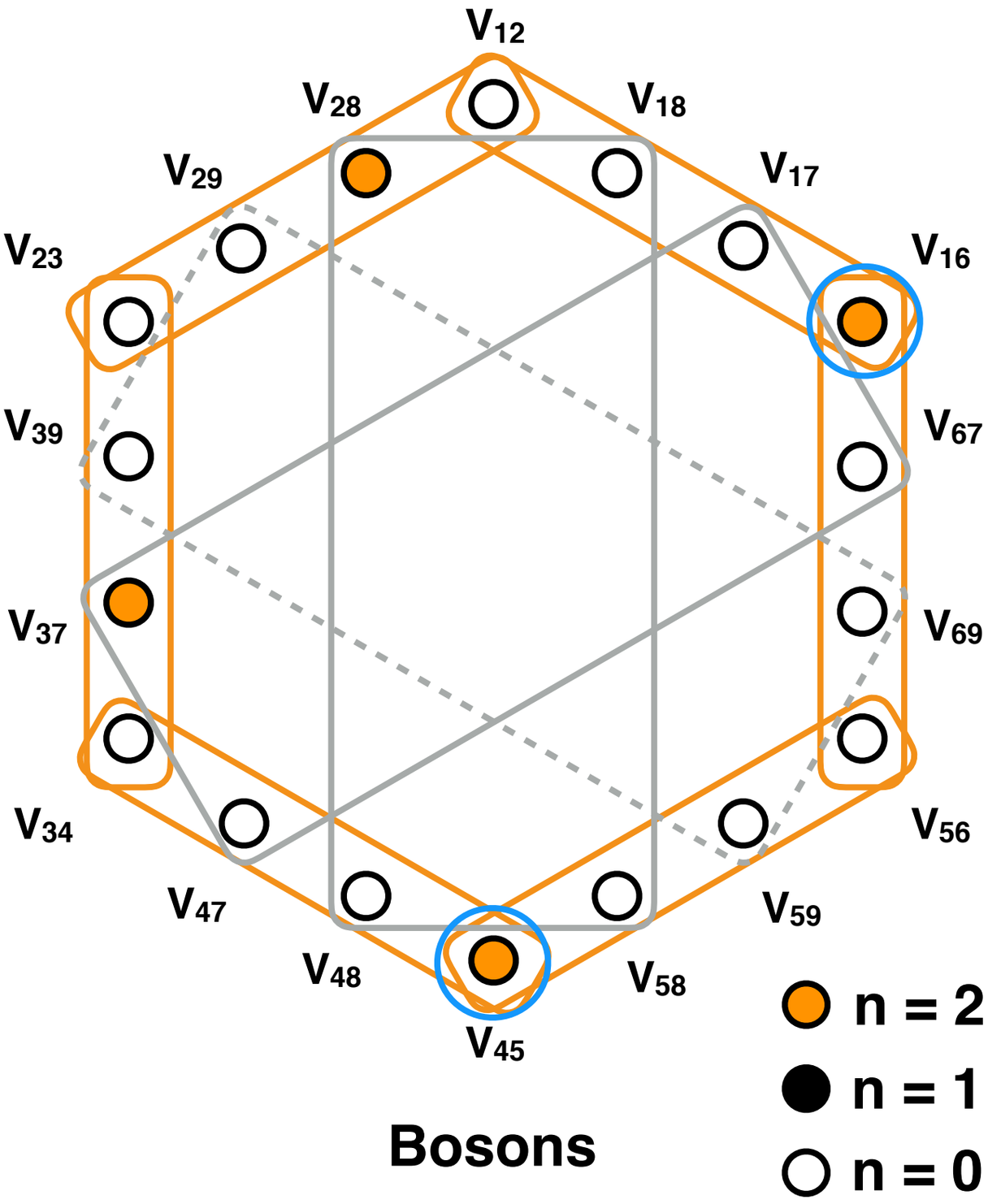



Fermions

$$|\psi\rangle = f^\dagger_{67} f^\dagger_{69} |0\rangle$$

$$|\psi\rangle = \left( \frac{f^\dagger_{39} f^\dagger_{23}}{2\sqrt{2}} + \frac{f^\dagger_{37} f^\dagger_{23}}{4} - \frac{f^\dagger_{37} f^\dagger_{39}}{4} - \frac{3 f^\dagger_{34} f^\dagger_{23}}{4} + \frac{f^\dagger_{34} f^\dagger_{39}}{4} - \frac{f^\dagger_{34} f^\dagger_{37}}{2\sqrt{2}} \right) |0\rangle$$

# Hardy-type contextuality - bosons




Bosons

$$|\psi\rangle = \frac{b_{16}^{\dagger 2}}{\sqrt{2}}|0\rangle$$

$$|\psi\rangle = \left( \frac{b_{45}^{\dagger 2}}{4\sqrt{2}} + \frac{b_{48}^{\dagger 2}}{4\sqrt{2}} - \frac{b_{47}^{\dagger 2}}{2\sqrt{2}} + \frac{b_{45}^{\dagger} b_{48}^{\dagger}}{4\sqrt{2}} - \frac{b_{45}^{\dagger} b_{47}^{\dagger}}{4} - \frac{b_{47}^{\dagger} b_{48}^{\dagger}}{4} \right) |0\rangle$$

# Conclusions and open problems

- **Indistinguishability restricts the set of measurements**
- **For more than one particle contextuality can be weaker**
- **In case of fermions the particle-hole symmetry is important**

- **Other dimensions and different number of modes**
- **Can one find examples for which there is no contextuality for N=1, but contextuality for N>1 ?**
- **What is the minimal contextual system for a given N ?**
- **...**